Third United Nations International Conference on the Peaceful Uses of Atomic Energy

A/CONF.28/A/527
CZECHOSLOVAKIA
February 1964
Submitted in:
ENGLISH and RUSSIAN

Confidential until official release during Conference

Abstract

SOME HEAT CONDUCTION THEORETICAL PROBLEMS IN REACTOR FUEL ELEMENTS

J. Schmid

A heat conduction boundary problem of nuclear reactor fuel elements with temperature dependent thermal conductivity and with nonlinear boundary conditions of the third and fourth type, i.e. with temperature dependent heat-transfer coefficient or with temperature dependent contact resistance on the interface of two media, is solved.

The heat transfer equation in cylindrical geometry

(1) $$\left[\frac{1}{r}\frac{\delta}{\delta r}\left(\lambda r\frac{\delta t}{\delta r}\right)+\frac{1}{r^2}\frac{\delta}{\delta\varphi}\left(\lambda\frac{\delta t}{\delta\varphi}\right)\right]=g(r,\varphi)$$

was transformed using the expression

(2) $$\lambda(t)=\lambda_o+\lambda_1(t)$$

into the form

(3) $$-\lambda_o\left[\frac{\delta^2 t}{\delta r^2}-\frac{1}{r}\frac{\delta t}{\delta r}+\frac{1}{r^2}\frac{\delta^2 t}{\delta\varphi^2}\right]=g(r,\varphi)+\lambda_1(t)\left[\frac{\delta^2 t}{\delta r^2}+\frac{1}{r}\frac{\delta t}{\delta r}+\frac{1}{r^2}\frac{\delta^2 t}{\delta\varphi^2}\right]+\frac{d\lambda_1}{dt}\left[\left(\frac{\delta t}{\delta r}\right)^2+\frac{1}{r^2}\left(\frac{\delta t}{\delta\varphi}\right)^2\right].$$

Supposing the contribution of the nonlinearities (i.e. the second and third terms on the right side of equation (3)) is small, we can find the solution using an iterative process. After rearrangement of the equation (3) we obtain

(4) $$\frac{\delta^2 t^{(m)}}{\delta r^2}+\frac{1}{r}\frac{\delta t^{(m)}}{\delta r}+\frac{1}{r^2}\frac{\delta^2 t^{(m)}}{\delta\varphi^2}=-g^{(m)}$$

where

$$g^{(o)}=\frac{1}{\lambda_o}\,g(r,\varphi) \qquad (m=o)$$

$$g^{(m)}=\frac{1}{\lambda_o}\left\{g(r,\varphi)-\lambda_1(t^{(m-1)})g^{(m-1)}+\frac{d\lambda_1}{dt}\Big|_{t=t^{(m-1)}}\left[\left(\frac{\delta t^{(m-1)}}{\delta r}\right)^2+\frac{1}{r^2}\left(\frac{\delta t^{(m-1)}}{\delta\varphi}\right)^2\right]\right\}$$

$$(m=1,2,3,\ldots).$$

The solution of the equation has the form

$$(5)\quad t^{(m)} = A_o^{(m)} + B_o^{(m)} \ell_{gr} + {}^{c}Z_o^{(m)} + \sum_{n=1}^{\infty} [(A_n^{(m)} r^n + B_n^{(m)} r^{-n} + {}^{c}Z_n^{(m)}) \cos n\varphi +$$

$$(C_n^{(m)} r^n + D_n^{(m)} r^{-n} + {}^{s}Z_n^{(m)}) \sin n\varphi]$$

where

$$\ell_{Z_n^{(m)}} = -r^n \int_{r_o}^{r} r'^{-(2n+1)} \int_{r_o'}^{r'} (r'')^{n+1} \ell_{\overline{g}_n^{(m)}}(r'')\, dr''\, dr' \qquad \ell = \begin{cases} c & (n=0,1,2,\ldots) \\ s & (n=1,2,3,\ldots) \end{cases}$$

and $\ell_{\overline{g}_m^{(m)}}(r)$ are the Fourier coefficients of the function $g^{(m)}$.

The boundary conditions (with small nonlinearities) can be expressed in a similar way; for example the boundary condition of the third type, which has the form

$$(6)\qquad -\lambda(t) \left.\frac{\delta t}{\delta r}\right|_{r=R} = \alpha(\varphi, t, t_R)\, [t(R,\varphi) - t_R(\varphi)]$$

can be written using relation (2) and the expression

$$\alpha(\varphi, t, t_R) = \phi(\varphi)[1 + F(t, t_R)]$$

as

$$(7)\quad \lambda_o \left.\frac{\delta t^{(m)}}{\delta r}\right|_{r=R} + \phi(\varphi)[t^{(m)}(R,\varphi) - t_R(\varphi)] =$$

$$-\phi(\varphi) F(t^{(m-n)}, t_R)\, [t^{(m-1)}(R,\varphi) - t_R(\varphi)] - \lambda_1(t^{(m-1)}) \left.\frac{\delta t}{\delta r}^{(m-1)}\right|_{r=R}$$

The solution of equation (7) can be found by a similar iterative process.

The boundary conditions varying along the perimeter were expanded in Fourier series and the boundary problems were solved using matrix calculation. External boundary conditions (the first, second, third ones) may be written in the form

$$_k\underline{M}\ \underline{C}^{(m)} = {}_k\underline{F}^{(m)} \qquad (k = 1,2,3)$$

and the conditions on the interface in the form

$$(8)\qquad \underline{C}_{\ell}^{(m)} = {}_4\underline{M}_{\ell-1,\ell}\ \underline{C}_{\ell-1}^{(m)} + {}_4\underline{F}_{\ell-1,\ell}^{(m)}$$

where $\underline{M}$ are matrixes, $\underline{F}$ vectors and $\underline{C}$ vectors whose components are integration constants A_o, B_o, A_1, B_1, C_1, D_1, ... The problem was solved also for axially

non-symetrical case (for a small eccentricity). The transformation of the co-ordinate system is of a similar form as the equation (8).

Solving the boundary problem for the annular region consisting of L annuli, it is necessary first to eliminate the integration constants of the intermediate annuli using the relation

$$\underline{C}_{\ell}^{(m)} = {}_4\underline{M}_{1,\ell}\,\underline{C}_1^{(m)} + {}_4\underline{F}_{1,\ell}^{(m)} \tag{9}$$

where

$${}_4\underline{M}_{1,\ell} = {}_4\underline{M}_{\ell-1,\ell} \cdot {}_4\underline{M}_{\ell-2,\ell-1} \cdots {}_4\underline{M}_{2,3} \cdot {}_4\underline{M}_{1,2}$$

$${}_4\underline{F}_{1,\ell}^{(m)} = {}_4\underline{M}_{\ell-1,\ell}\left\{{}_4\underline{M}_{\ell-2,\ell-1}\left[{}_4\underline{M}_{\ell-3,\ell-2}(\ldots) + {}_4\underline{F}_{\ell-3,\ell-2}^{(m)}\right] + {}_4\underline{F}_{\ell-2,\ell-1}^{(m)}\right\} + {}_4\underline{F}_{\ell-1,\ell}^{(m)}$$

and then to solve the equation system

$$\begin{cases} k_1\underline{M}_1\,\underline{C}_1^{(m)} = k_1\underline{F}_1^{(m)} \\ k_L\underline{M}_L\,\underline{C}_L^{(m)} = k_L\underline{F}_L^{(m)} \\ \underline{C}_L^{(m)} = {}_4\underline{M}_{1,L}\,\underline{C}_1^{(m)} + {}_4\underline{F}_{1,L}^{(m)} \end{cases} \tag{10}$$

After eliminating $\underline{C}_L^{(m)}$ we solve $\underline{C}_1^{(m)}$ and then $\underline{C}_{\ell}^{(m)}$ using the expression (9).

It may be expected that the iterative process will converge, if non-linearities caused by respecting the temperature dependence of the thermal conductivity, the heat-transfer coefficient and the contact resistance are small.

If the thermal conductivity, the heat-transfer coefficient and the contact resistance are temperature independent, it is sufficient to find the solution of the equation (4) for the given boundary conditions as the first step of the iterative process.

The method was applied to some problems in calculation of hot channel factors.